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THE INFLUENCE OF ASPECT RATIO AND MATERIAL ON THE SOUND RADIATION OF LOW FREQUENCY MODES OF BAFFLED CYLINDRICAL SHELLS

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1. INTRODUCTION

Circular cylindrical shells are parts of many machines and industrial plants; often, their sound radiation strongly influences the sound radiated by the whole machine. The radiation efficiency of a cylindrical shell depends on the acoustic properties of shell modes.

Here only flexural modes of vibration are considered. If the circular cylindrical shell is simply supported they are expressed by the equation [1]:

$$u = A\sin(m\pi x/L)\cos(n(\theta - \phi)) \tag{1}$$

where x is the axial co-ordinate, θ the angular co-ordinate, u the radial displacement, $m = 1, 2, \ldots$ the axial mode order, and $n = 0, 1, 2, \ldots$ the circumferential mode order; L is the cylinder length, and A and ϕ are two constants.

Some typical plots of natural frequencies against mode order are shown in Figures 1 and 2; modal density is low only in the frequency bands which contain the low frequency modes.

In the frequency bands with high modal density the radiation efficiency can be calculated by a statistical approach [2]. In the frequency bands of the low frequency modes the radiation properties of the single modes are important. Some interesting questions are: what is the influence of shell shape and material on the efficiency of low frequency modes? are there any combinations of these parameters which minimize radiation efficiency?

The aim in this Letter to the Editor is to answer these questions. An analytical approach is used and complicated equations were manipulated and solved with software used for algebraic manipulation [3, 4].

2. ELEMENTS OF THE ACOUSTICS OF CYLINDRICAL SHELLS

The geometric properties of a simply supported cylindrical shell are defined by two aspect ratios: $X = a/L$ and $Y = h/a$, where a, L and h are cylinder radius, length and thickness respectively. The material properties are defined by the ratio $Z = E/\rho$ of Young's modulus and the density, and by the Poisson ratio μ (which does not vary significantly for the range of metals of industrial interest).

Two structural wavenumbers can be defined for a mode of order m, n of a finite length cylinder: the circumferential wavenumber and the axial wavenumber.

Since there is no edge effect in the circumferential direction, the wavenumber transform of the mode in this direction shows only one line at the circumferential wavenumber $k_s = n/a$. On the other hand, there is an edge effect in the axial direction and the wavenumber transform shows a main peak centred on the axial wavenumber $k_z = m\pi/L$ and side peaks of lower amplitude. The structural wavenumbers can be expressed in a

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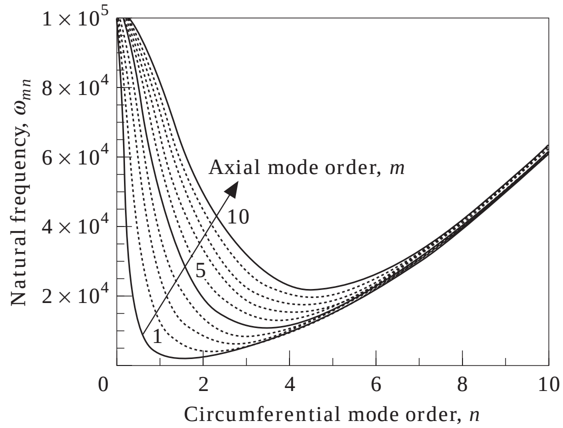

Figure 1. The natural frequencies of the modes of a steel cylinder with radius $a = 0{\cdot}05$ m, thickness $h = 0{\cdot}001$ m, length $L = 0{\cdot}8$ m.

non-dimensional way by multiplying them by the cylinder radius and $K_s = n$, $K_z = m\pi a/L = m\pi X$ are obtained.

The radiation efficiency of a mode of order m, n excited in resonance conditions depends on wavenumbers K_s, and K_z, and on the non-dimensional wavenumber of sound $K_a = \omega_{mn} a/c$, where c is the sound speed in air and ω_{mn} is the natural frequency which is equal to the frequency of excitation.

The following radiation conditions are possible: condition 1, $K_z > K_a$ and $K_s > K_a$: the mode is subsonic in both directions and the radiation efficiency is very low; condition 2, $K_z < K_a$ and $K_s > K_a$: the main peak of the axial wavenumber transform is supersonic, whereas the circumferential wavenumber is subsonic; the whole cylinder surface radiates like an n-pole [5] and the radiation efficiency is low; condition 3, $K_z^2 + K_s^2 < K_a^2$: the mode is supersonic and has a high radiation efficiency [2, 5]; condition 4, $K_z > K_a$ and $K_s < K_a$: the mode is subsonic in the axial direction and supersonic in the circumferential direction and the radiation efficiency is low; only the edges of the cylinder radiate (edge mode [2]).

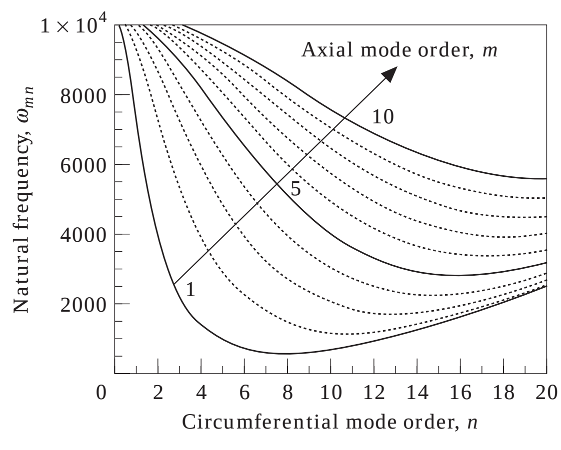

Figure 2. The natural frequencies of the modes of a steel cylinder with radius $a = 0{\cdot}5$ m, thickness $h = 0{\cdot}001$ m, length $L = 1$ m.

If $m = 1$, the wavenumber transform in axial direction peaks at $K_z = 0$ and the main peak is always supersonic; therefore only conditions 2 and 3 are relevant. If $n = 0$, the mode in the circumferential direction behaves like a monopole (breathing mode).

In order to understand the influence of cylinder aspect ratios and material on radiation properties of low frequency modes, first the structural wavenumbers of these modes have to be found, and then the equations which define the different conditions of radiation have to be expressed in terms of X, Y and Z.

An analytical formula for the natural frequency is necessary since it appears in the acoustic wavenumber. The equation of Soedel [1], which is appropriate for $n \neq 0$ modes of simply supported cylindrical shells and is based on the Donnell–Mushtari–Vlasov equation, was chosen. The equation is written in terms of parameters X, Y and Z as

$$\omega = \frac{\sqrt{\{K_z^4/(K_s^2 + K_z^2)^2\} + \frac{1}{12}\{(K_s^2 + K_z^2)^2 Y^2/(1-\mu^2)\}}\sqrt{Z}}{a}. \qquad (2)$$

3. THE INFLUENCE OF CYLINDER ASPECT RATIOS ON THE ORDER OF LOW FREQUENCY MODES

The two diagrams of natural frequencies against mode order (Figures 1 and 2) clearly show that, for low values of m, there is a value of n which minimizes natural frequency; this value is named $n_{min}(m)$ and depends on m and on cylinder aspect ratios. The lowest natural frequency is always associated to the mode with $m = 1$ and $n_{min}(1)$. The modes with $m = 1$, 2 and n close to $n_{min}(m)$ have low frequencies and the maximum probability of belonging to the low modal density region. Therefore the wavenumbers $n_{min}(1)$ and $n_{min}(2)$ give a clear indication of the circumferential wavenumbers of low frequency modes.

Szechenyi [2] has given a formula for determining the lowest natural frequency for any value of m and for $n \geq 2$. This formula was written in terms of X, Y and Z as

$$\omega_{min}(m) = \frac{\sqrt{2}\pi X\sqrt{Y}\sqrt{Z}m}{12^{0\cdot25}(1-\mu^2)^{0\cdot25}a}. \qquad (3)$$

The circumferential wavenumber $K_s = n_{min}(m)$ which minimizes natural frequency for any value of m was found by equating equations (2) and (3):

$$\frac{\sqrt{2}\pi X\sqrt{Y}\sqrt{Z}m}{12^{0\cdot25}(1-\mu^2)^{0\cdot25}a} = \frac{\sqrt{\{K_z^4/(K_s^2 + K_z^2)^2\} + \frac{1}{12}\{K_s^2 + K_z^2)^2 Y^2/(1-\mu^2)\}}\sqrt{Z}}{a}. \qquad (4)$$

There is an eighth degree equation in K_s. The solution which has physical significance is

$$n_{min}(m) = \sqrt{\frac{3^{0\cdot25}\sqrt{2}(1-\mu^2)^{0\cdot25}\pi X m}{\sqrt{Y}} - \pi^2 X^2 m^2}. \qquad (5)$$

This equation is rather simple and shows that neither cylinder radius nor Z influence $n_{min}(m)$.

Solution (5) was plotted for $m = 1$ and $m = 2$ against X and Y and the contour plots of Figures 3 and 4 were obtained; these plots are of practical interest since the designer can enter the aspect ratios and calculate $n_{min}(m)$. Wavenumber n is an integer; if the point of co-ordinates X and Y is in the region between two contour lines, the order of the mode which minimizes natural frequency is that of the closest contour line.

The contour plot for $m = 1$ shows that for pipes and long cylinders (X small) $2 \leqslant n_{min} \leqslant 5$. If the length of the cylinder is equal to the diameter ($X = 0\cdot5$), n_{min} is higher, especially if the wall is thin. n_{min} is very high only for short cylinders (X large) with little thickness (Y *small*). Similar conclusions can be drawn from the contour plot for $m = 2$.

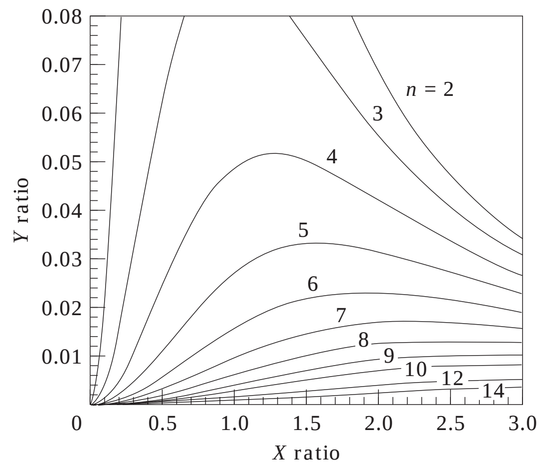

Figure 3. Steel cylinders: the circumferential wavenumber of the mode $m = 1$ with minimum frequency.

4. THE INFLUENCE OF CYLINDER ASPECT RATIOS AND MATERIAL ON THE RADIATION OF LOW FREQUENCY MODES

The equations which describe the transitions among the radiation conditions were written in terms of X, Y and Z:

$$K_z = K_a \Rightarrow Y_z = \frac{2\sqrt{3}\sqrt{1-\mu^2}\pi m X\sqrt{\pi^4 c^2 m^4 X^4 - \pi^2 Z m^2 X^2 + 2\pi^2 c^2 m^2 n^2 X^2 + c^2 n^4}}{\sqrt{Z}(\pi^4 m^4 X^4 + 2\pi^2 m^2 n^2 X^2 + n^4)}, \quad (6)$$

$$K_s = K_a \Rightarrow Y_s = \frac{2\sqrt{3}\sqrt{1-\mu^2}\sqrt{-\pi^4 Z m^4 X^4 + \pi^4 c^2 m^4 n^2 X^4 + 2\pi^2 c^2 m^2 n^4 X^2 + c^2 n^6}}{\sqrt{Z}(\pi^4 m^4 X^4 + 2\pi^2 m^2 n^2 X^2 + n^4)}, \quad (7)$$

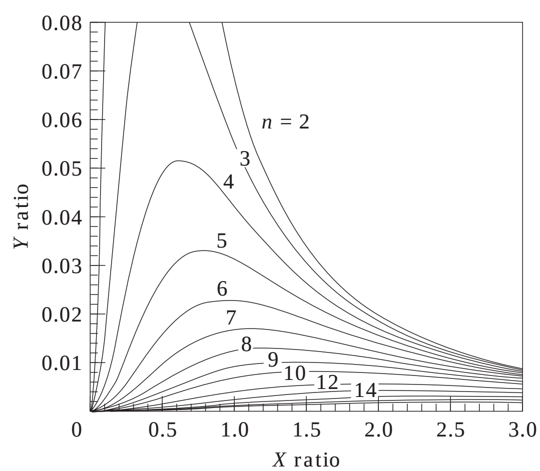

Figure 4. Steel cylinders: the circumferential wavenumber of the mode $m = 2$ with minimum frequency.

$$K_z^2 + K_s^2 = K_a^2$$

$$\Rightarrow Y_{zs} = \frac{2\sqrt{3}\sqrt{1-\mu^2}\sqrt{\pi^6 c^2 m^6 X^6 + (3c^2 n^2 - Z)\pi^4 m^4 X^4 + 3\pi^2 c^2 m^2 n^4 X^2 + c^2 n^6}}{\sqrt{Z}(\pi^4 m^4 X^4 + 2\pi^2 m^2 n^2 X^2 + n^4)}. \quad (8)$$

For assigned values of mode order and material properties these equations, which do not depend on cylinder radius, divide the X–Y plane into the regions represented in Figure 5. In region 1 condition 1 is satisfied, in region 2 condition 2 is satisfied, and so on. There are transition zones between subsonic and supersonic conditions in which $K_z < K_a$ and $K_s < K_a$, but $K_z^2 + K_s^2 > K_a^2$.

Plots such as those in Figure 5 help in understanding how the acoustic behaviour of the m, n mode depends on the aspect ratios. Figure 5 refers to steel cylinders ($Z = 2{\cdot}624 \times 10^7\ \mathrm{m^2/s^2}$, $\mu = 0{\cdot}3$) and shows that condition 1 and condition 4 can be satisfied only if Y is lower than a maximum value ($Y_M = 0{\cdot}0074$) which is the same for both conditions. The maximum value Y_M, if n is low, does not depend on mode order and is expressed by

$$Y_M = \sqrt{3}c^2\sqrt{1-\mu^2}/Z. \quad (9)$$

The comparison between Figures 5(a) and 5(b) shows that if n increases (or decreases) by one, there are not large modifications of radiation properties.

Figure 6 is relevant to the same modes as those in Figure 5, but the cylinder is made of bronze ($Z = 1{\cdot}332 \times 10^7\ \mathrm{m^2/s^2}$, $\mu = 0{\cdot}35$). The areas of regions 1, 2 and 4 are larger for a bronze cylinder than for a steel cylinder, and therefore there are more possibilities of having low radiation efficiency; in particular the extensions of regions 1 and 4 in Y direction are increased and the Y_M for a bronze cylinder is 0·0143.

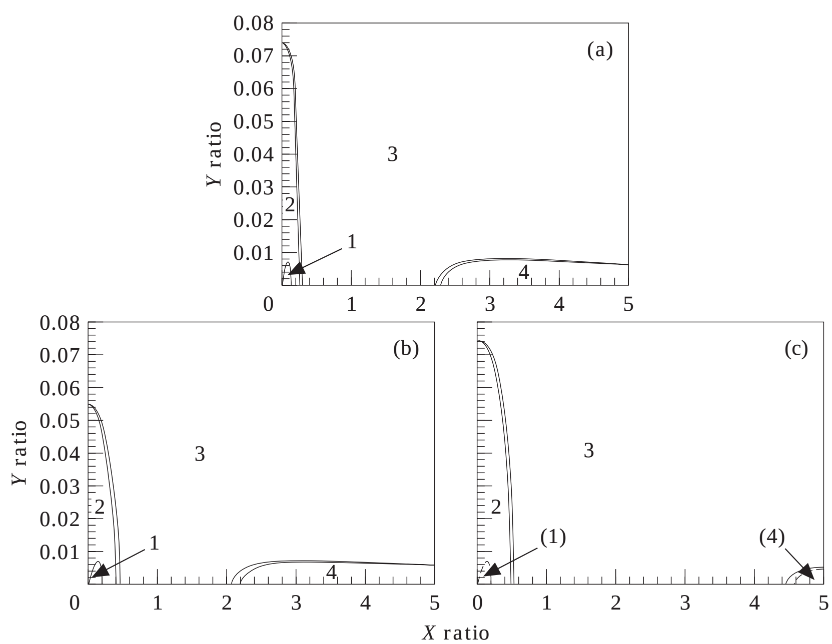

Figure 5. Steel cylinders: the acoustic behaviour of modes (a) $m = 2$, $n = 3$, (b) $m = 2$, $n = 4$ and (c) $m = 1$, $n = 3$.

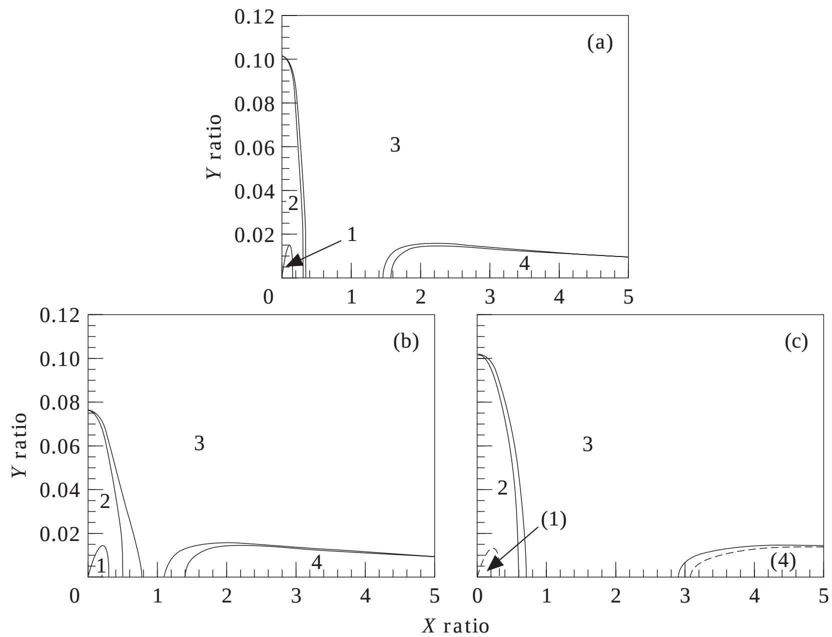

Figure 6. Bronze cylinders: the acoustic behaviour of modes (a) $m = 2$, $n = 3$, (b) $m = 2$, $n = 4$ and (c) $m = 1$, $n = 3$.

For aluminium cylinders $Z = 2{\cdot}652 \times 10^7$ m^2/s^2, $\mu = 0{\cdot}34$. Hence the plots which showed the radiation properties of the mode against X, Y are very similar to those for steel cylinders ($Y_M = 0{\cdot}0072$).

For steel and aluminium cylinders Y_M is very small, and therefore the mode is subsonic in both directions (condition 1) or behaves like an edge mode (condition 4) only if the cylinder is very thin. For thicker cylinders, there is only one transition between low radiation efficiency (region 2) and high radiation efficiency (region 3); therefore in the following considerations only this transition will be dealt with.

The acoustic properties of the single mode are of interest only if the mode has a low frequency; therefore the most important information about radiation efficiency has to be

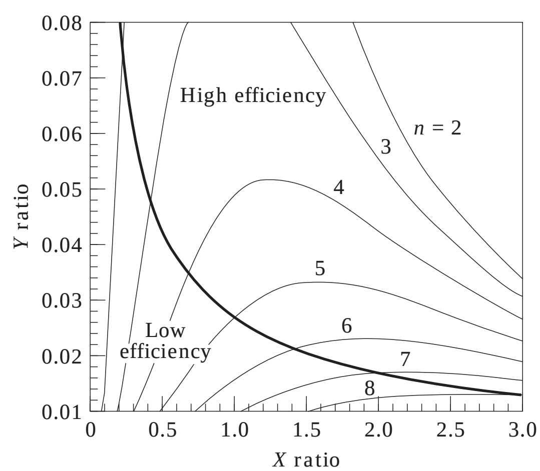

Figure 7. Steel cylinders: the acoustic behaviour of the mode $m = 1$ with minimum frequency.

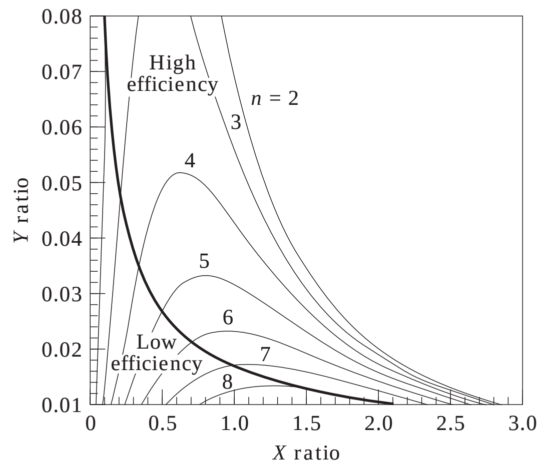

Figure 8. Steel cylinders: the acoustic behaviour of the mode $m = 2$ with minimum frequency.

introduced in the contour plots (Figures 3 and 4). These plots for each pair X, Y allow the mode of axial order m that has minimum frequency to be identified. In equation (8), which defines the transition between low and high radiation efficiency, m was established and n was set equal to n_{min} and the following equation was obtained:

$$Y = (3^{0\cdot 75}\sqrt{2}(1 - \mu^2)^{0\cdot 75}c^2/\pi XZm)^{2/3}. \tag{10}$$

This equation defines the points of the X–Y plane where the transition between low radiation efficiency happens for the mode of order m which has minimum frequency. Equation (10) was then plotted together with the contour plots which show the order of the minimum frequency mode, and the plots of Figures 7 and 8, which relate to steel cylinders with $Y > 0{\cdot}01$, were obtained. These plots summarize the acoustic behaviour of the low frequency modes of cylinders since, for given values of aspect ratios, they show whether the mode of axial order m and minimum frequency has high or low radiation efficiency. Long steel cylinders with $X < 0{\cdot}5$ and little thickness $Y = 0{\cdot}01 \div 0{\cdot}03$ radiate with low efficiency if they are excited in the low frequency range.

5. CONCLUSIONS

This study of the acoustic properties of low frequency modes of cylinders has shown that for steel and aluminium cylinders the most important transition is that between condition 2 (subsonic circumferential wavenumber) and condition 3 (supersonic mode). For cylinders of other materials (e.g. bronze) condition 1 (subsonic mode) and 4 (edge mode) are more common.

A series of formulas and plots that help one to understand the radiation efficiency of low frequencies modes has been presented.

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